UKMT Team Maths Challenge 2012

Poster Competition

In addition to four rounds similar to those at the regional finals (Group Circus, Crossnumber, Head-to-Head and Relay), there will be a Poster Competition at the National Final. All teams are required to submit a poster. This will be judged separately and will not affect the Team Maths Challenge score, but forms an integral part of the National Final.

After the competition some posters may be retained by the UKMT in order to be reproduced for promotional purposes; all the original posters will eventually be returned to schools.

On the day, teams will have 50 minutes to create a poster on a sheet of A1 paper (landscape), which will be provided. Sheets of A4 paper will also be available.

The subject of the poster will be *Mathematical impossibility* (see overleaf). Teams must carry out research into this topic in the weeks leading up to the final.

Teams may create materials beforehand, but such prepared work must be on sheets no larger than A4 and must be assembled to create the poster on the day.

A team which arrives with a poster already assembled will be disqualified.

The materials of the poster must not extend beyond the edge of the A1 paper.

The judges will not touch the poster, so all information must be clearly visible.

Your team number (assigned to you on arrival) must be clearly visible in the bottom right-hand corner of the poster. There must be nothing else on the poster to identify the team.

Reference books may not be used at the competition, and large extracts copied directly from books or the internet will not receive much credit.

Teams must bring with them any drawing equipment they think they will need.

Glue sticks and scissors will be provided.

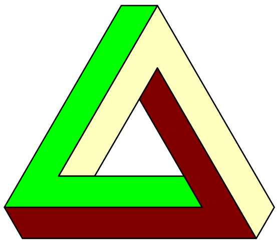
The content of each poster is limited only by the imagination of the team members. *However, on the day each team will be presented with three questions on the subject—the answers to these questions must be incorporated into the structure of the poster*. Teams may be asked to provide geometric or algebraic proofs, and some ingenuity may be involved.

Posters will be judged on the following criteria:

General mathematical content	(12 marks)
Imagination and presentation	(12 marks)
Answers to the questions	(24 marks)

Mathematical impossibility

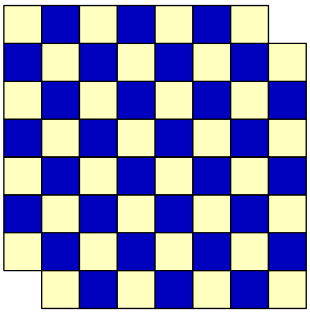
An *impossible object* is a type of optical illusion consisting of a two-dimensional figure which appears to represent a three-dimensional object, even though it is not geometrically possible for such an object to exist. A well known example is the *Penrose triangle*, shown on the right.

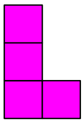


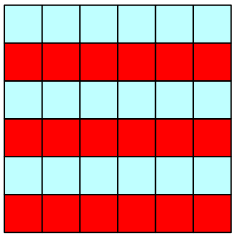
> Why is the Penrose triangle an impossible object?

In mathematics, it may be possible to *prove* that something is impossible.

Example 1 *Suppose we remove two squares from opposite corners of an* 8×8 *chessboard (see the left-hand figure below). Then it is not possible to tile the remaining board with* 1×2 *dominoes, without gaps or overlaps.*







To prove the result in example 1, observe that the remaining board has more squares of one colour than the other. However, no matter how a domino is placed it will cover one square of each colour. Therefore any number of dominoes will only ever cover an equal number of squares of each colour, so that it is not possible to cover the remaining board.

This is an example of a *parity*, or *colouring*, proof.

Example 2 *The L-tetromino is the shape shown in the centre above, made from four connected* 1×1 *squares. A* 6×6 *board cannot be tiled with L-tetrominoes.*

> The right-hand figure above shows a colouring of the 6×6 board. How can this be used to give a parity proof of the result in example 2?

A *knight* is a chess piece which moves two squares in one direction ('horizontally' or 'vertically') and one square on the other direction. The complete move therefore looks like the letter 'L'. A *knight's tour* is a sequence of moves visiting every square of the board exactly once. A *closed tour* is one where the last square visited is a knight's move away from the starting square.

Example 3 *A closed knight's tour on a* 4×4 *board is impossible.*

> Is there a colouring proof of the result in example 3?